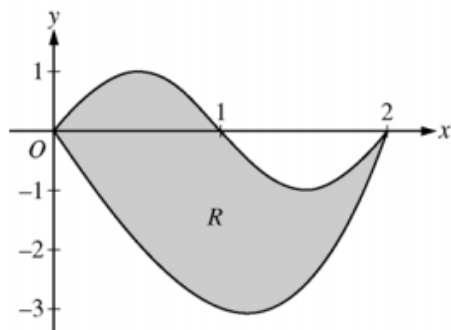


Area and Volume Question Styles

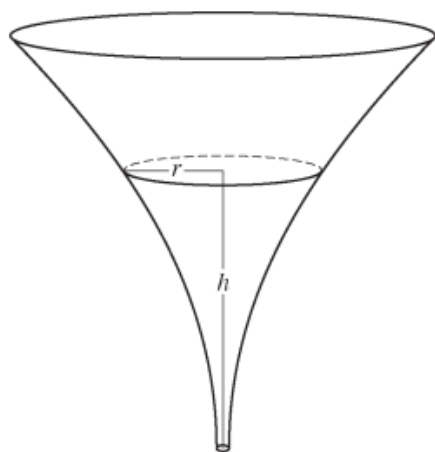
1.

A graphing calculator is required for some problems or parts of problems.



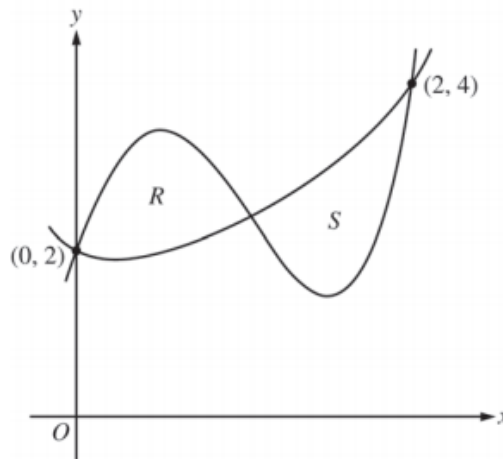
1. Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.
- Find the area of R .
 - The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.
 - The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a square. Find the volume of this solid.
 - The region R models the surface of a small pond. At all points in R at a distance x from the y -axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

2.



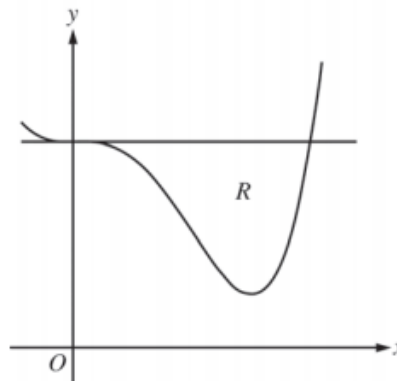
5. The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h , the radius of the funnel is given by $r = \frac{1}{20}(3 + h^2)$, where $0 \leq h \leq 10$. The units of r and h are inches.
- Find the average value of the radius of the funnel.
 - Find the volume of the funnel.

3.



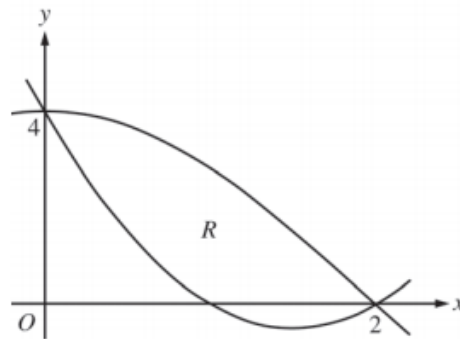
2. Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2-2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.
- Find the sum of the areas of regions R and S .
 - Region S is the base of a solid whose cross sections perpendicular to the x -axis are squares. Find the volume of the solid.

4.



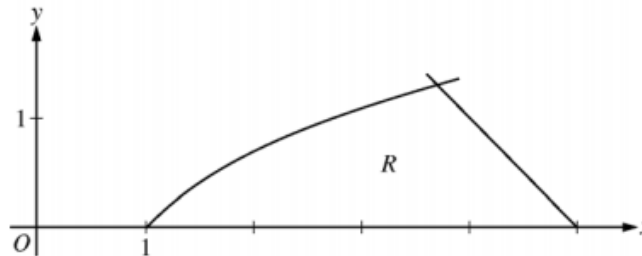
2. Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.
- Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.
 - Region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is an isosceles right triangle with a leg in R . Find the volume of the solid.
 - The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k .

5.



5. Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g , as shown in the figure above.
- Find the area of R .
 - Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.
 - The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

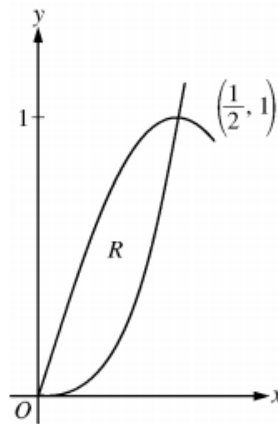
6.



2. Let R be the region in the first quadrant bounded by the x -axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.
- Find the area of R .
 - Region R is the base of a solid. For the solid, each cross section perpendicular to the x -axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.
 - The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k .

7.

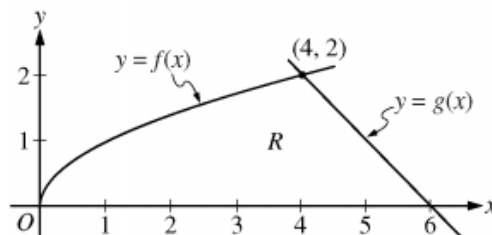
No calculator is allowed for these problems.



3. Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.
- Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.
 - Find the area of R .
 - Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

8.

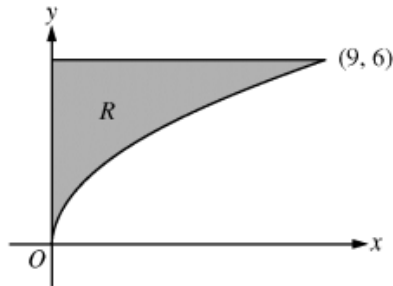
No calculator is allowed for these problems.



3. The functions f and g are given by $f(x) = \sqrt{x}$ and $g(x) = 6 - x$. Let R be the region bounded by the x -axis and the graphs of f and g , as shown in the figure above.
- Find the area of R .
 - The region R is the base of a solid. For each y , where $0 \leq y \leq 2$, the cross section of the solid taken perpendicular to the y -axis is a rectangle whose base lies in R and whose height is $2y$. Write, but do not evaluate, an integral expression that gives the volume of the solid.
 - There is a point P on the graph of f at which the line tangent to the graph of f is perpendicular to the graph of g . Find the coordinates of point P .

9.

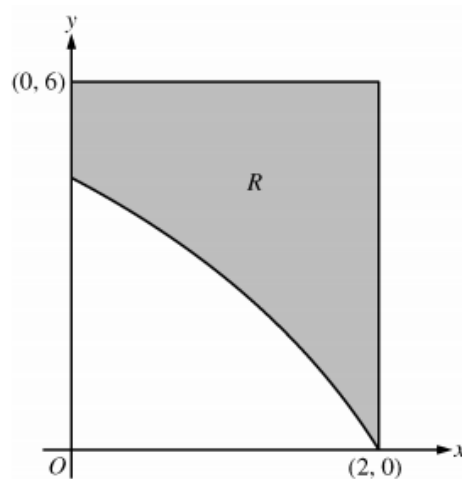
No calculator is allowed for these problems.



4. Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y -axis, as shown in the figure above.
- Find the area of R .
 - Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.
 - Region R is the base of a solid. For each y , where $0 \leq y \leq 6$, the cross section of the solid taken perpendicular to the y -axis is a rectangle whose height is 3 times the length of its base in region R . Write, but do not evaluate, an integral expression that gives the volume of the solid.

10.

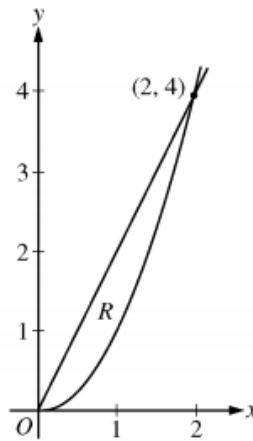
A graphing calculator is required for some problems or parts of problems.



1. In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3-x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.
- Find the area of R .
 - Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.
 - The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a square. Find the volume of the solid.

11.

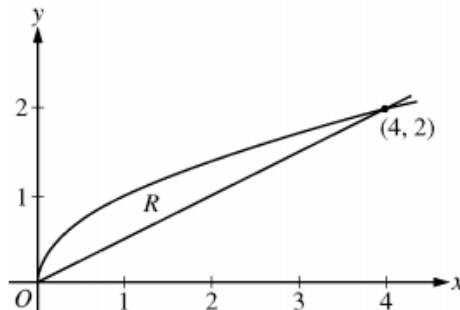
No calculator is allowed for these problems.



4. Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.
- Find the area of R .
 - The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x -axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.
 - Another solid has the same base R . For this solid, the cross sections perpendicular to the y -axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

12.

No calculator is allowed for these problems.



4. Let R be the region bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{2}$, as shown in the figure above.
- Find the area of R .
 - The region R is the base of a solid. For this solid, the cross sections perpendicular to the x -axis are squares. Find the volume of this solid.
 - Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 2$.

13.

A graphing calculator is required for some problems or parts of problems.

- Let R be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$.
 - Find the area of R .
 - Find the volume of the solid generated when R is rotated about the vertical line $x = -1$.
 - The region R is the base of a solid. For this solid, the cross sections perpendicular to the y -axis are squares. Find the volume of this solid.

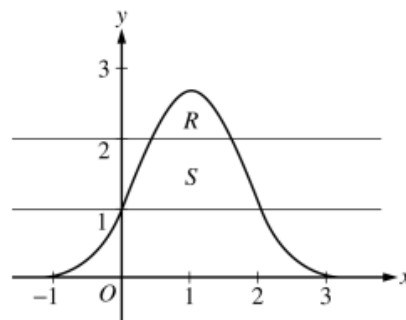
14.

A graphing calculator is required for some problems or parts of problems.

- Let R be the region in the first and second quadrants bounded above by the graph of $y = \frac{20}{1+x^2}$ and below by the horizontal line $y = 2$.
 - Find the area of R .
 - Find the volume of the solid generated when R is rotated about the x -axis.
 - The region R is the base of a solid. For this solid, the cross sections perpendicular to the x -axis are semicircles. Find the volume of this solid.

15.

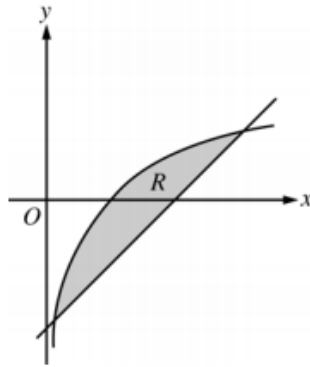
A graphing calculator is required for some problems or parts of problems.



- Let R be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2$, and let S be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.
 - Find the area of R .
 - Find the area of S .
 - Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

16.

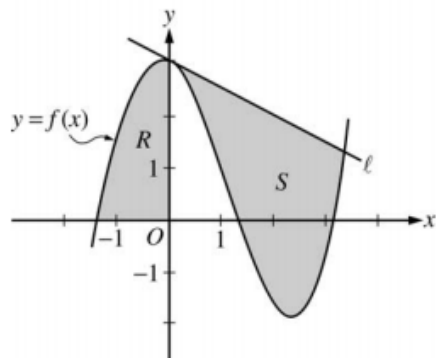
A graphing calculator is required for some problems or parts of problems.



1. Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$, as shown above.
 - (a) Find the area of R .
 - (b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.
 - (c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about the y -axis.

17.

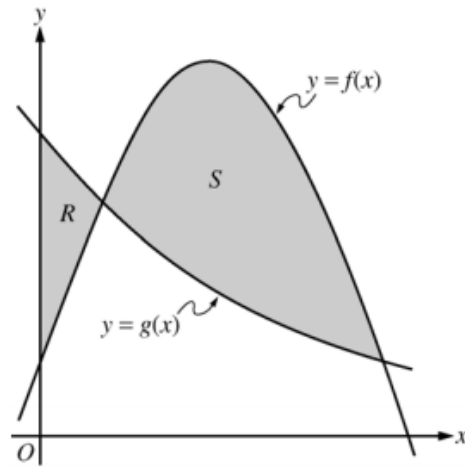
A graphing calculator is required for some problems or parts of problems.



1. Let f be the function given by $f(x) = \frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x$. Let R be the shaded region in the second quadrant bounded by the graph of f , and let S be the shaded region bounded by the graph of f and line ℓ , the line tangent to the graph of f at $x = 0$, as shown above.
 - (a) Find the area of R .
 - (b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.
 - (c) Write, but do not evaluate, an integral expression that can be used to find the area of S .

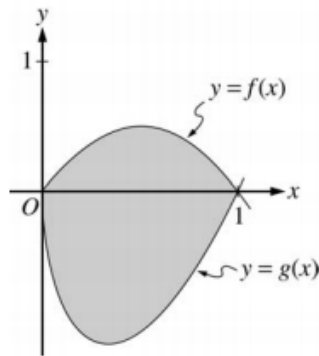
18.

A graphing calculator is required for some problems or parts of problems.



1. Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the shaded region in the first quadrant enclosed by the y -axis and the graphs of f and g , and let S be the shaded region in the first quadrant enclosed by the graphs of f and g , as shown in the figure above.
- Find the area of R .
 - Find the area of S .
 - Find the volume of the solid generated when S is revolved about the horizontal line $y = -1$.

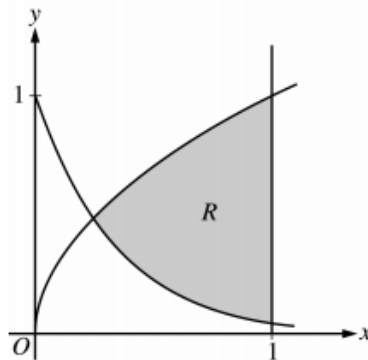
19.



2. Let f and g be the functions given by $f(x) = 2x(1 - x)$ and $g(x) = 3(x - 1)\sqrt{x}$ for $0 \leq x \leq 1$. The graphs of f and g are shown in the figure above.
- Find the area of the shaded region enclosed by the graphs of f and g .
 - Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 2$.
 - Let h be the function given by $h(x) = kx(1 - x)$ for $0 \leq x \leq 1$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x -axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k .

20.

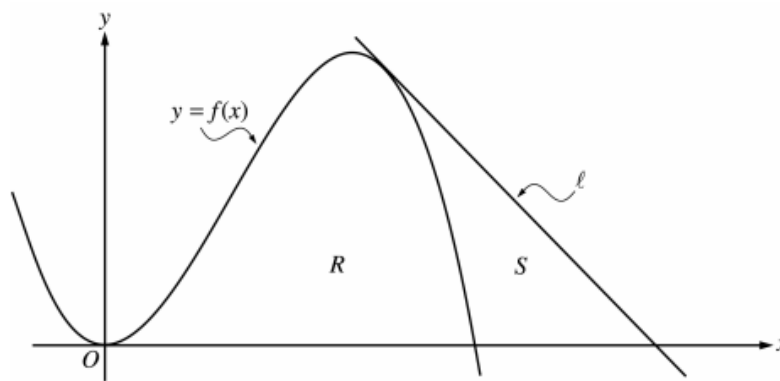
A graphing calculator is required for some problems or parts of problems.



- Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.
 - Find the area of R .
 - Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.
 - The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a rectangle whose height is 5 times the length of its base in region R . Find the volume of this solid.

21.

A graphing calculator is required for some problems or parts of problems.



1. Let f be the function given by $f(x) = 4x^2 - x^3$, and let ℓ be the line $y = 18 - 3x$, where ℓ is tangent to the graph of f . Let R be the region bounded by the graph of f and the x -axis, and let S be the region bounded by the graph of f , the line ℓ , and the x -axis, as shown above.
- Show that ℓ is tangent to the graph of $y = f(x)$ at the point $x = 3$.
 - Find the area of S .
 - Find the volume of the solid generated when R is revolved about the x -axis.