D05 Sample Assessment Questions

See D05 Practice QQ file (discussed in class).

Also …

TEXTBOOK 3.6

- Exercises 3, 5, 6, 22, 23, 24, 25, 28, 30.

KHAN ACADEMY:

- Derivatives of Inverse Functions
- Derivatives of Inverse Trigonometric Functions
- Differentiate Logarithmic Functions

TEXTBOOK 3.7:

- Exercises: 2, 3, 4, 9, 13, 15, *16, 23, 27, 31, 33.
- Desmos graphs of the 3.7 Exercise functions - THESE GRAPHS ARE FUN!

KHAN ACADEMY

- Implicit Differentiation

as well as the following:

SOLUTIONS for the above problems are given elsewhere on the Husky Hub and within Khan Academy.

Solutions for problems below are given at the end of this handout.)

1. $g(x) = \arctan(e^x)$.

Find the following:

(a) $g(0)$

(b) the instantaneous rate of change of function g at $x = 0$:

(c) an equation for the tangent line to $y = g(x)$ at $x = 0$:

2.

Figure 3.30 shows $f(x)$ and $f^{-1}(x)$. Using Table 3.6, find

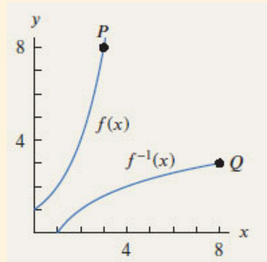

Figure 3.30

x	$f(x)$	$f'(x)$
0	1	0.7
1	2	1.4
2	4	2.8
3	8	5.5

Table 3.6

(a)

(i) $f(2)$

(ii) $f^{-1}(2)$

(iii) $f'(2)$

(iv) $(f^{-1})'(2)$

(b) The equation of the tangent lines at the points P and Q.

(c) What is the relationship between the two tangent lines?

SOLUTIONS

1. $g(x) = \arctan(e^x)$.

Find the following:

(a) $g(0) = \arctan(e^0) = \arctan(1) = \pi/4$

(b) the instantaneous rate of change of function g at $x = 0$:

$$g'(x) = \left(\frac{1}{1+\left(e^x\right)^2}\right)\left(e^x\right) = \frac{e^x}{1+e^{2x}}$$

$$g'(0) = \frac{e^0}{1+e^0} = \frac{1}{2}$$

(c) an equation for the tangent line to $y = g(x)$ at $x = 0$:

$$y - \frac{\pi}{4} = \frac{1}{2}(x - 0) \quad \text{or} \quad y = 0.5x + 0.25\pi$$

2.

(a) Reading from the table, we have

(i) $f(2) = 4$.

(ii) $f^{-1}(2) = 1$.

(iii) $f'(2) = 2.8$.

(iv) To find the derivative of the inverse function, we use

$$(f^{-1})'(2) = \frac{1}{f'(f^{-1}(2))} = \frac{1}{f'(1)} = \frac{1}{1.4} = 0.714.$$

Notice that the derivative of f^{-1} is the reciprocal of the derivative of f. However, the derivative of f^{-1} is evaluated at 2, while the derivative of f is evaluated at 1, where $f^{-1}(2) = 1$ and $f(1) = 2$.

(b) At the point P, we have $f(3) = 8$ and $f'(3) = 5.5$, so the equation of the tangent line at P is

$$y - 8 = 5.5(x - 3).$$

At the point Q, we have $f^{-1}(8) = 3$, so the slope at Q is

$$(f^{-1})'(8) = \frac{1}{f'(f^{-1}(8))} = \frac{1}{f'(3)} = \frac{1}{5.5}.$$

Thus, the equation of the tangent line at Q is

$$y - 3 = \frac{1}{5.5}(x - 8).$$

(c) The two tangent lines have reciprocal slopes, and the points (3, 8) and (8, 3) are reflections of one another across the line $y = x$. Thus, the two tangent lines are reflections of one another across the line $y = x$.