APCalculus - Rentz - Content Assessment (CON) - 30 minutes

| START TIME: STOP TIME: | Last Name $\qquad$ <br> Name Called By $\qquad$ <br> Period: (circle one) BLUE <br> Class: 2019(SR) 2020(JR) <br> Date: $\qquad$ | PD 4 2021(SOPH) | SCORE <br> 0 $\qquad$ 3 <br> $/ 11 \mathbf{p t s}$ <br> \% |
| :---: | :---: | :---: | :---: |
| NO CALCULATOR FOR THIS STANDARD! |  |  |  |
| Show your work! |  |  |  |
| Honor Code Reminders: |  |  |  |
| Do your own work. |  |  |  |
| No collaboration. |  |  |  |
| Do not discuss this assessment with other students prior to one week after the assessment UNLESS the teacher discusses details in class before that time. |  |  |  |

## Do your best!

(Finding Definite Integrals: The Basics)

1. Use the graph of $y=f(x)$ below to find the requested values. Show work.

Note: Graph consists of a semicircle and 3 line segments.

(a) $[+1 \mathrm{pt}] \quad \int_{0}^{2} f(x) d x=0$
(b) $[+1 \mathrm{pt}] \quad \int_{6}^{8} f(x) d x$

$$
\text { Trapezoid! }(1 / 2)(2+4)(2)=6
$$

(c) $[+1 \mathrm{pt}] \quad \int_{2}^{8} f(x) d x$

Area(rectangle) - Area(semicircle) +Trapezoid in part (b)

$$
\begin{aligned}
& =(2)(4) \quad-(1 / 2)\left(\pi(2)^{2}\right)+6 \\
& =14-2 \pi
\end{aligned}
$$

(d) $[+1 \mathrm{pt}] \quad \int_{8}^{10} f(x) d x$

$$
\begin{aligned}
& \text { Toughest one! } \\
& \text { Equation for line is } y-4=(-3)(x-8) \\
& \mathrm{x} \text {-intercept }(\mathrm{y}=0) \quad-4=(-3)(\mathrm{x}-8) \\
& -4-24=-3 x \\
& x=28 / 3=91 / 3
\end{aligned}
$$

$$
(1 / 2)(4)(4 / 3)-(1 / 2)(2)(2 / 3)=(8 / 3)-(2 / 3)=6 / 3=2
$$

2. $\int_{a}^{b} f(x) d x=55$.

$$
\int_{a}^{b} g(x) d x=20
$$

Find each of the following exactly. Show work to justify results.
(a) $[+\mathbf{1} \mathbf{p t}]$

$$
\begin{aligned}
& \int_{a}^{b}(f(x)-g(x)) d x \\
= & \int_{a}^{b} f(x) d x-\int_{a}^{b} g(x) d x \\
= & 55-20=35
\end{aligned}
$$

(b) $[+\mathbf{1} \mathbf{p t}] \quad \int_{a}^{b}(-2 g(x)) d x$

$$
=-2 \int_{a}^{b} g(x) d x=-2(20)=-40
$$

(c) $[+\mathbf{1} \mathbf{~ p t}] \quad \int_{b}^{a}(f(x)) d x$

$$
=-\int_{a}^{b} f(x) d x=-(55)=-55
$$

(d) $[+\mathbf{1} \mathbf{p t}] \quad$ If $c$ is any real number, $\int_{a}^{c}(g(x)) d x+\int_{c}^{b}(g(x)) d x$.

$$
=\int_{a}^{b} g(x) d x=(20)
$$

(e) $[+\mathbf{1} \mathrm{pt}] \quad \int_{a}^{b}(2 f(x)+3 g(x)) d x$

$$
\begin{aligned}
& =2 \int_{a}^{b} f(x) d x+3 \int_{a}^{b} g(x) d x \\
& =2(55)+3(20)=110+60=170
\end{aligned}
$$

3. [ $+1 \mathrm{pt}]$ Evaluate, using the graph of $\boldsymbol{y}=\boldsymbol{f}(\boldsymbol{x})$ below: $\int_{0}^{5} f(x) d x$ if it exists.
(If the definite integral does not exist, write DNE.)


If a "mostly" continuous function has only "jump" and "removable" discontinuities, then a definite integral can be found:

$$
\begin{aligned}
& =\int_{0}^{2}(1) d x+\int_{2}^{4}(2) d x+\int_{4}^{5}(2) d x \\
& =(1)(2)+2(2)+(2)(1)=2+4+2=8
\end{aligned}
$$

4. $[+1 \mathrm{pt}] \quad$ Write the definite integral that provides the answer to the following question: (Since you do not have access to your calculator for this assessment, you do not need to evaluate your definite integral.)


Let $R$ be the shaded region in the first quadrant enclosed by the $y$-axis and the graphs of $y=1-x^{3}$ and $y=\sin \left(x^{2}\right)$, as shown in the figure above.
(a) Find the area of $R$.

$$
=\int_{a}^{c}\left(1-x^{3}\right) d x-\int_{a}^{c} \sin \left(x^{2}\right) d x
$$

If $c$ is the first coordinate of the intersection point.

