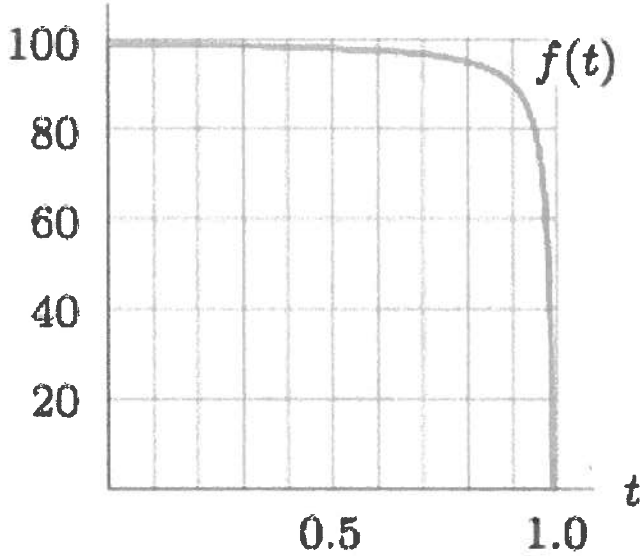
1. No Calculator

KEY

The graph of $f(t)$ is in Figure 5.34. Which of the following four numbers could be an estimate of $\int_0^1 f(t)dt$ accurate to two decimal places? Explain your choice.

I. -98.35 II. 71.84 III. 100.12 $\boxed{\text{IV. } 93.47}$



a bit less than 1.

Figure 5.34

2. No Calculator

Use Figure 5.36 to find the values of each of the following:

(a) $\int_a^b f(x)\,dx$ = 13 **(b)** $\int_b^c f(x)\,dx$ = −2

(c) $\int_a^c f(x)\,dx$ = 11 **(d)** $\int_a^c |f(x)|\,dx$ = 13+2 = 15

13 +(−2)

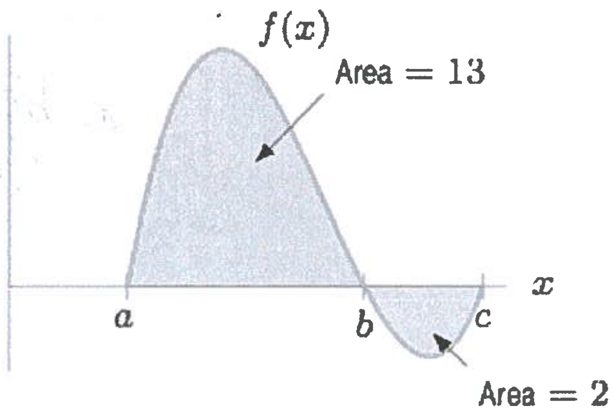
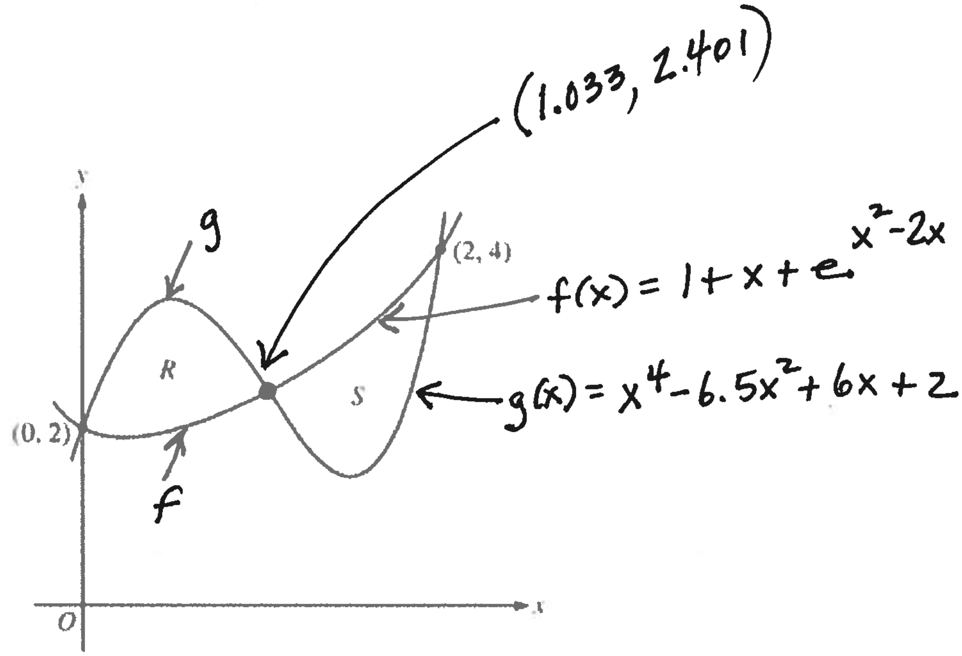


Figure 5.36

3. Calculator



Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2-2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.

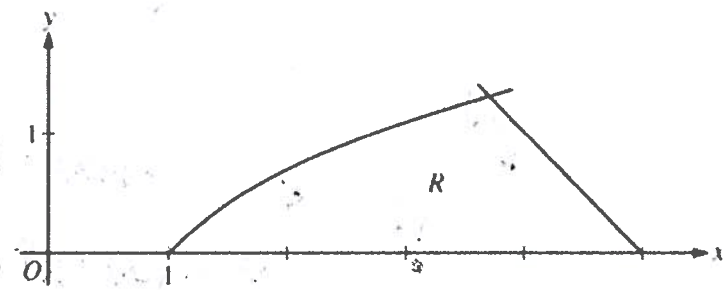
(a) Find the sum of the areas of regions R and S.

$$\int_0^{1.033} (g(x)-f(x))\,dx + \int_{1.033}^{2} (f(x)-g(x))\,dx$$

$$0.997 + 1.007$$

$$2.004$$

4. Calculator



Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.

The HORIZONTAL LINE y = k divides R into two regions of equal area.
Find the value of k.

HINT:
This problem is easier if you use the inverse functions x = ln(y) and x = 5 - y.

$\int_0^{1.307} \left((5-x)-e^x\right)dx \approx 2.986$ area of R

$\int_0^{k} (5-x-e^x)\,dx \approx 2.986/2$

$\left(5x - \frac{x^2}{2} - e^x\right)\Big|_0^k \approx \cancel{2.986}\ 1.493$

$\left(5k - \frac{k^2}{2} - e^k\right) - \left(0 - 0 - e^0\right) \approx \cancel{2.986}\ 1.493$

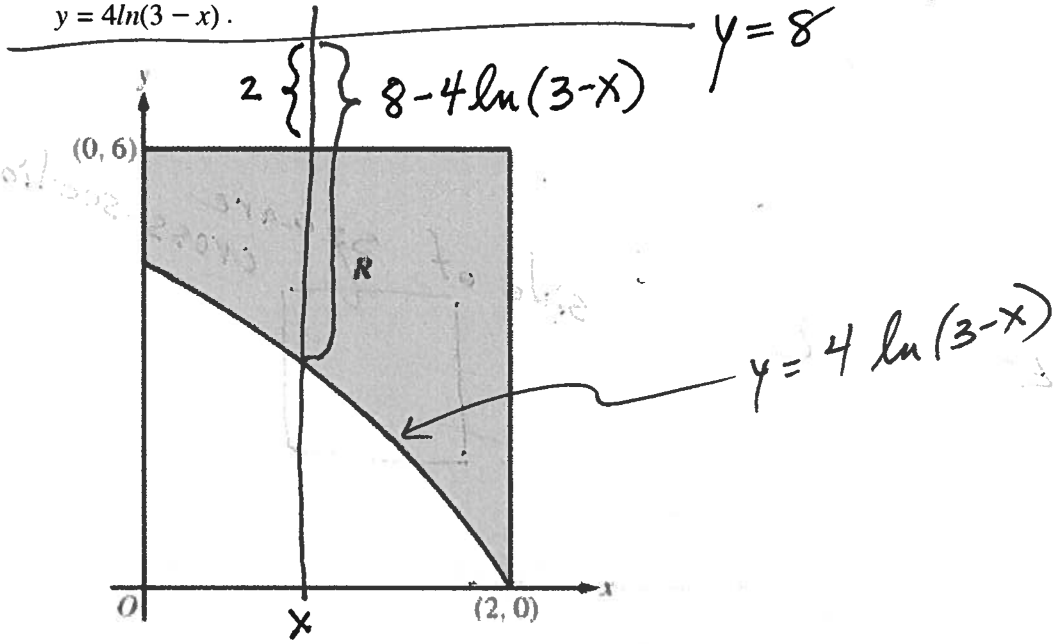
$5k - \frac{1}{2}k^2 - e^k + 1 \approx \cancel{2.986}\ 1.493$

$5x - \frac{1}{2}x^2 - e^x \cancel{2.986} - 0.493 \approx 0$

Use calculator

$\boxed{x \approx \cancel{1.30}\ 0.421}$

5. The WHITE REGION in the diagram is created by the x-axis and y-axis and the curve $y = 4ln(3 - x)$.



Region R is formed by vertical lines $x = 0$ and $x = 2$, horizontal line $y = 6$, and the curved line $y = 4\ln(3-x)$.

(a) Find the volume of the solid created when region R is revolved around the x-axis.

(b) Find the volume of the solid generated when region R is revolved around the line $y = 8$.

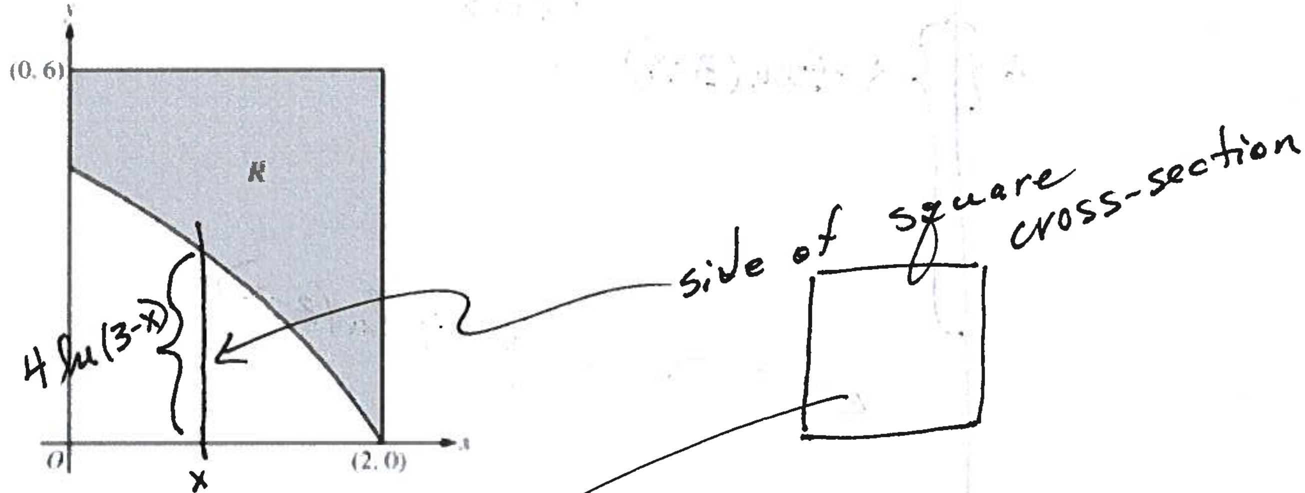
(a) $$\int_0^2 \left(\pi(6)^2 - \pi\left(4\ln(3-x)\right)^2\right) dx$$

$$\int_0^2 \left(36\pi - \pi\left(4\ln(3-x)\right)^2\right) dx \approx \boxed{174.463}$$

(b) $$\int_0^2 \left(\pi\left(8-4\ln(3-x)\right)^2 - \pi(2)^2\right) dx$$

$$\int_0^2 \left(\pi\left(8-4\ln(3-x)\right)^2 - 4\pi\right) dx \approx \boxed{168.180}$$

6.



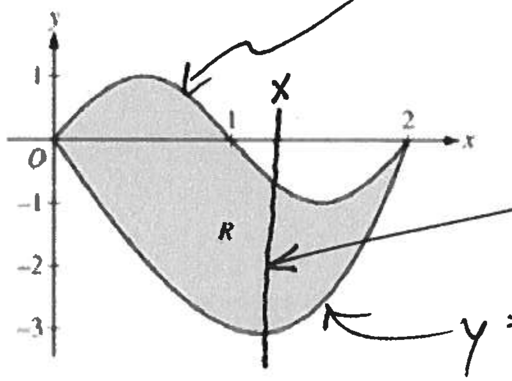
The **WHITE REGION** in the diagram is created by the x- and y-axes and the curve $y = 4\,ln(3 - x)$. Imagine a solid with the white region as its base. For every x value in the interval [0,2], a cross-section of the solid (sliced perpendicular to the x-axis) has the shape of a square. Find the volume of this "solid with known cross-section."

area

$$\int_0^2 (4\ln(3-x))^2 \, dy \approx \boxed{16.467}$$

7.

$y = \sin(\pi x)$



x

length of rectangle: $\sin(\pi x) - (x^3 - 4x)$

$y = x^3 - 4x$

Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above. The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$.

Write, <u>but do not evaluate</u>, an expression involving integrals that gives the volume of water in this pond.

cross-section is a rectangle

depth of Water at x $(3-x)$

cross section

area of cross-section:

$$(3-x)(\sin(\pi x) - x^3 + 4x)$$

Volume is $\boxed{\int_0^2 (3-x)(\sin(\pi x) - x^3 + 4x)\,dx}$