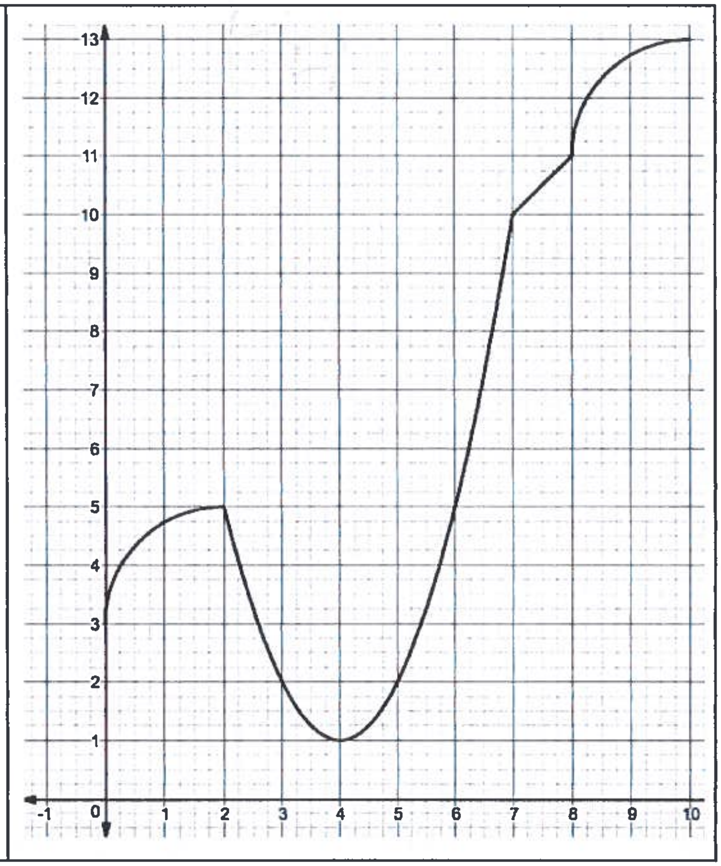
Standard 11 Problems

You may use a calculator in answering these questions. **[22 points]**

Use this function f and its graph at right for questions 1-8.

$$f(x)=\begin{cases}\sqrt{-x^2+4x}+3 & 0\le x<2\\ x^2-8x+17 & 2\le x<7\\ x+3 & 7\le x<8\\ \sqrt{-x^2+20x-96}+11 & 8\le x\le 10\end{cases}$$



[+1] 1. Give all x values in (0,10) where f ' (x) = 0:

4

[+1] 2. Give all x values in (0,10) where f ' (x) is undefined:

2, 7, 8

[+1] 3. Give all sub-intervals of (0,10) where function f is decreasing:

(2, 4)

[+1] 4. Give all sub-intervals of (0,10) where function f is concave up:

(2, 7)

$\boxed{+4}$

[+4]5. Give the x-values for each of the following:

(a) all local (relative) maximum points:

2,

In choosing local extrema, we do not consider end points.
In choosing global extrema we do consider end points

(b) all global (absolute) maximum points:

10

(c) all global (absolute) minimum points:

4

(d) all critical points:

2, 4, 7, 8

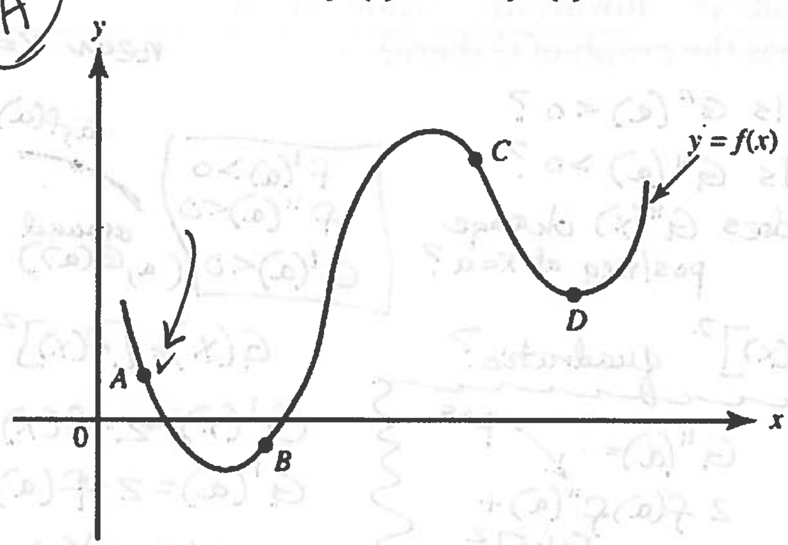
[+2]6. Is (2,5) a point of inflection? Explain why or why not.

Yes
CD to CU

[+1] 7. At which one point (A, B, C, or D) on the following graph of $y = f(x)$ is the following statement true:

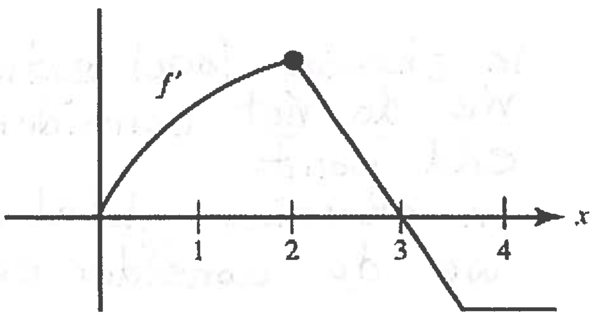
A

$$f'(x) < 0 \text{ and } f''(x) > 0\ ?$$



$\boxed{+7}$

[+1] 8. D



The graph of f', which consists of a quarter-circle and two line segments, is shown above. At $x = 2$ which of the following statements is true?

~~(A)~~ f is not continuous. it's differentiable ⇒ continuous

~~(B)~~ f is continuous but not differentiable. There's a value for $f'(2)$

~~(C)~~ f has a local maximum. at $x=3$ not $x=2$ (at $x=3$, $f'(3)=0$)

(D) The graph of f has a point of inflection. changes concavity

Note: slope of tangent line to $y=f'(x)$ changes from positive to negative. at $x=2$

* [+1] 9. B

Let $G(x) = [f(x)]^2$. In an interval around $x = a$, the graph of f is increasing and concave downward, while G is decreasing. Which describes the graph of G there? near $x=a$

NO (A) concave downward Is $G''(a)<0$?

→ (B) concave upward Is $G''(a)>0$?

(C) point of inflection does $G''(x)$ change pos/neg at $x=a$?

(D) quadratic NO Is $[f(x)]^2$ quadratic?

$\boxed{\begin{array}{l} f'(a)>0 \\ f''(a)<0 \\ G'(a)<0 \end{array}}$

$(a, f(a))$

around $(a, G(a))$ decreasing

$G''(a) = 2f(a)f''(a) + 2[f'(a)]^2$

pos

pos

>0

$G(x) = [f(x)]^2$

$G'(x) = 2 \cdot f(x) \cdot f'(x)$

$G'(a) = 2 \cdot f(a) \cdot f'(a)$

$2 \cdot f(a) \cdot f'(a) < 0$

pos NEG! pos $\boxed{f(a)<0}$

Note:
Give approx. to 3 decimal places!

[+9] 10. Free-Response (Calculator OK)

A function f is defined on the interval [0,4], and its derivative is $f'(x) = e^{\sin x} - 2\cos 3x$.

(+3) (a) On what interval is f increasing? Justify your answer.

(+3) (b) At what value(s) of x does f have local maxima? Justify your answer.

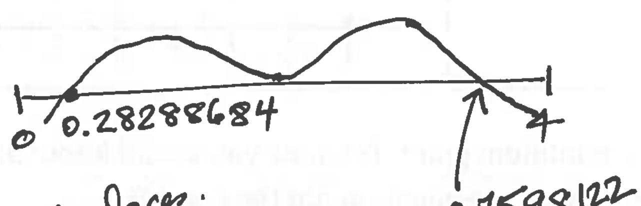
(+3) (c) How many points of inflection does the graph of f have? Justify your answer.

$$f''(x) = e^{\sin x}(\cos x) - 2(-\sin 3x \cdot 3)$$
$$= e^{\sin x}(\cos x) + 6\sin 3x$$

$$f'(x) = e^{\sin x} - 2\cos 3x$$

(a) Where is $f'(x) > 0$? f is increasing iff $f' > 0$.

Graph $y = f'(x)$ on $[0,4]$ to get



at least 3 decimal places.

$\boxed{(0.283, 3.760)}$ Interval

$\boxed{f' > 0.\ f \text{ increasing}}$ Reason

(b) $\boxed{x = 3.7598}$ value

local max since you're going from increasing before to decreasing after — 1st Deriv Test

reason- 1st or 2nd deriv. test by name!

$f''(3.7598) = -6.229038 < 0 \to CD \to$ local max — 2nd Deriv Test

Reason

(c) Find where f'' is 0 or undefined. Then, check to see if concavity changes.

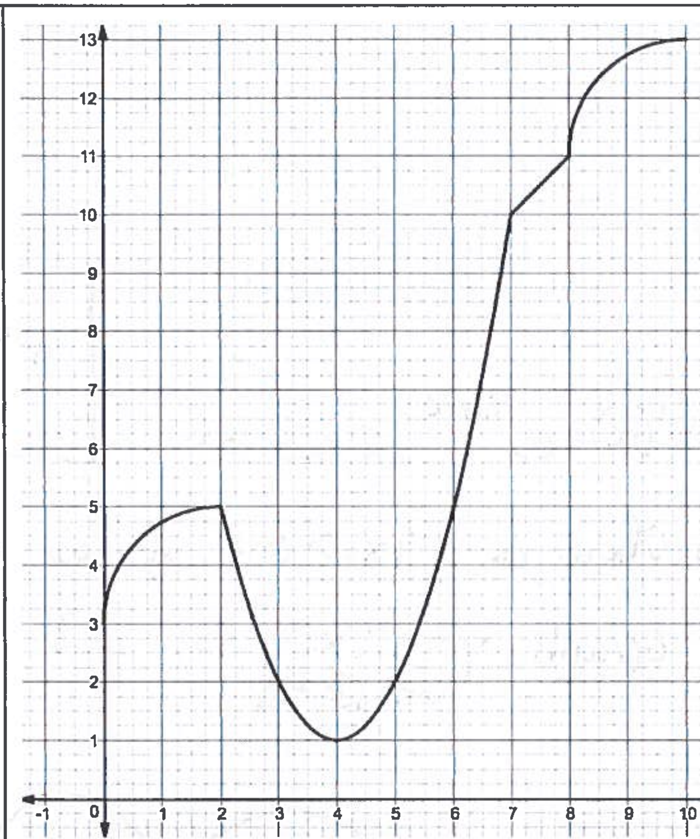
\# points $\boxed{3}$ graph of $y = f''$ is continuous and has 3 zeroes on $[0,4]$. At each zero, Concavity changes.

Standard 12 Problems

DO NOT USE A CALCULATOR ON THIS SECTION. SHOW WORK! **[20 points]**

Use this function f and its graph at right for questions 1 and 2.

$$f(x)=\begin{cases}\sqrt{-x^2+4x}+3 & 0\le x<2\\ x^2-8x+17 & 2\le x<7\\ x+3 & 7\le x<8\\ \sqrt{-x^2+20x-96}+11 & 8\le x\le 10\end{cases}$$



[+2] 1. Point (4,1) is a local minimum point. Tell how you would know this by using the <u>first derivative test</u>. (Note: Rely on the equation <u>not the graph</u>!)

at (4,1) $f(x)=x^2-8x+17 \rightarrow f'(x)=2x-8$

$f'(4)=0$

Check $f'(c)$ for c values before and after 4. $\rightarrow$ $\begin{cases} f'(x)<0 \text{ to left. } f \text{ decreasing} \\ f'(x)>0 \text{ to right. } f \text{ increasing}\end{cases}$

By 1st deriv. test $\rightarrow$ local minimum point.

(f' changes from negative to positive at $x=4$.)

[+2] 2. Point (4,1) is a local minimum point. Tell how you would know this by using the <u>second derivative test</u>. (Note: Rely on the equation <u>not the graph</u>!)

See #1 $f'(x)=0$ if $x=4$. $f'(x)$ is never undefined.

$f''(x)=2>0$

$f''(4)>0$

By 2nd deriv test $\rightarrow$ local minimum point.

$\boxed{f'(4)=0 \text{ and } f''(4)>0 \rightarrow \text{local min}}$

[+2] 3. Show work and do NOT use a calculator.

B

The derivative of a function f is given for all x by

$$f'(x) = \underbrace{x^2}_{\text{pos}}(x+1)^3\underbrace{(x-4)^2}_{\text{pos}}.$$

The set of x values for which f is a relative minimum is

(A) $\{0, -1, 4\}$

$\boxed{\text{(B) } \{-1\}}$ $\boxed{+2 \text{ for B}}$

(C) $\{0, 4\}$

(D) $\{0, -1\}$

$f'(x) = 0$ iff $x = 0, -1, 4$

f': <0 | -1: 0 | >0 | 0: 0 | >0 | 4: 0 | >0

$-2 \quad -1 \quad 0 \quad 4$

-1: min; 0: max; 4: neither

Note:
At $x=0$, derivative does not change from positive to negative!
At $x=4$ derivative does not change from positive to negative!

[+2] 4. Show work and do NOT use a calculator.

The maximum value of the function $f(x) = xe^{-x}$ is

$\boxed{\text{(A) } \frac{1}{e}}$ y value

(B) 1 ← x value

(C) -1 no

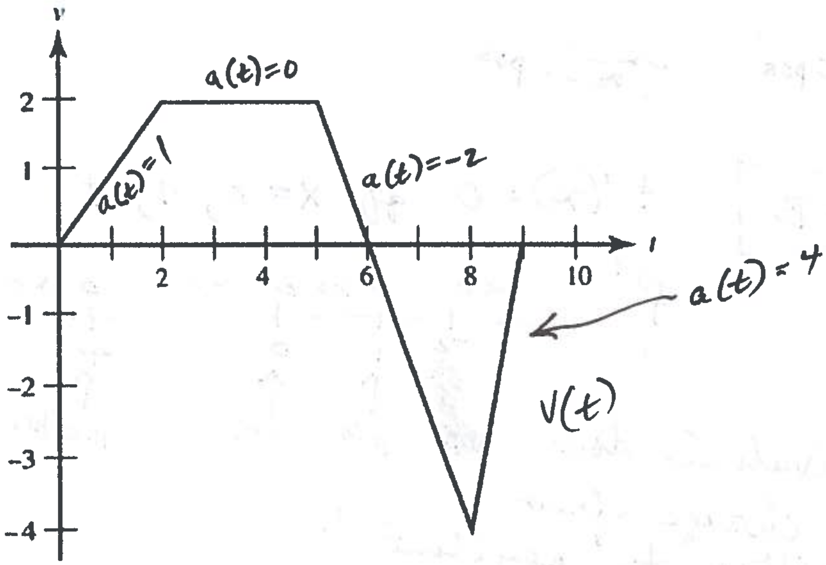
(D) $-e$ no

$\boxed{\begin{array}{l} +1 \text{ for B} \\ +2 \text{ for A} \end{array}}$

$$f'(x) = x \cdot e^{-x}(-1) + e^{-x}(1)$$
$$= e^{-x} - xe^{-x}$$
$$= e^{-x}(1-x) \quad \text{always defined}$$
$$= 0 \text{ iff } 1 - x = 0$$
$$x = 1$$
$$f(1) = 1 \cdot e^{-1} = \frac{1}{e}$$

[+1] 5. D

The graph below shows the <u>velocity</u> of an object moving along a line for $0 \le t \le 9$.



At what time(s) does the object attain its maximum acceleration?

(A) $2 < t < 5$

(B) $t = 6$

(C) $t = 8$

(D) $8 < t < 9$

$a(t) = v'(t)$

[+2] 6. D **Show work and do NOT use a calculator!**

The value of c for which $f(x) = x + \frac{c}{x}$ has a local minimum at $x = 3$ is

(A) -9

(B) 0

(C) 6

(D) 9

$$f'(x) = 1 + c\left(-\frac{1}{x^2}\right)$$
$$= 1 - \frac{c}{x^2}$$
$$f'(3) = 1 - \frac{c}{3^2} = 0$$
$$1 - \frac{c}{9} = 0$$
$$\frac{c}{9} = 1$$
$$c = 9$$