Standard N05 N06 FORM A
(Basic Applications of The Integral & The Fundamental Theorem of Calculus II)

KEY

APCalculus - Rentz - Content Assessment (CON) - 30 minutes

START TIME:	Last Name ____________	SCORE
	Name Called By ____________	0 ______ 6
	Period: (circle one) BLUE PD 4	__ / 17 pts
STOP TIME:	Class: 2019(SR) 2020(JR) 2021(SOPH)	___ %
	Date: ____________	

NO CALCULATOR FOR THIS STANDARD!

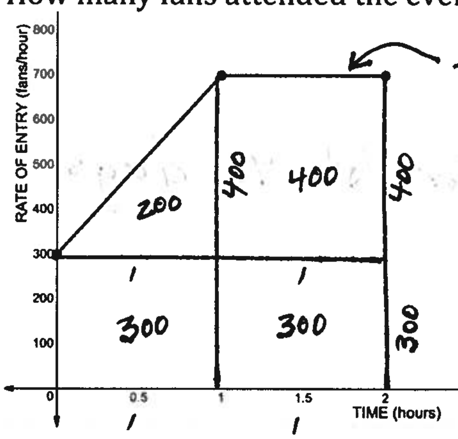
Show your work!

Honor Code Reminders:
Do your own work.
No collaboration.
Use only the technology approved by the teacher for this assessment.
Do not discuss this assessment with other students prior to one week after the assessment UNLESS the teacher discusses details in class before that time.

Do your best!

SECTION ONE: General Applications of The Integral Concept

[+1] 1. A sports arena opens 2 hours before the start of an event.
The graph below shows the <u>rate at which fans enter the arena</u> as a function of time.
How many fans attended the event?



function f

$\int_0^2 f(x)\,dx =$

area under curve

200
300
400
300

$\boxed{1200 \text{ fans}}$

+1

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[+1] 2. Write an expression that would help you answer the following question easily if you could use a calculator. You do not need to find the final solution in a simplified form.

The water level in a waterway is changing at a rate of $\frac{4}{3}\sin\left(2-\frac{t}{2}\right)$ centimeters per hour (where t is the number of hours since midnight). By approximately how many centimeters does the water level change between $t = 1$ and $t = 5$?
(i.e. what is the net change in water level between 1:00 am and 5:00 am?)

$\boxed{\underbrace{\int_1^5 \left(\frac{4}{3}\sin\left(2-\frac{t}{2}\right)\right)dt}_{\text{into calculator}}}$ or $\underbrace{\left(\frac{8}{3}\cos\left(2-\frac{t}{2}\right)\right)\Big|_1^5}_{\text{requires u-substitution Method!}}$ or approx 2.152 (if calculated)

[+1] 3. Write an expression that would help you to answer the following question easily if you could use a calculator. You do not need to find the final solution in a simplified form.

The depth of the water in a bird bath is changing at a rate of $r(t) = 0.25t - 0.1$ millimeters per hour (where t is the time in hours). At time $t = 0$, the depth of the water is 35 millimeters. What is the depth of the water at $t = 3$ hours?

(amount at time $t=0$) + (changed amount $t=0$ to $t=3$)

$35 + \int_0^3 r(t)\,dt$, or $\boxed{35 + \int_0^3 (0.25t-0.1)\,dt}$

or $35 + \left(\frac{t^2}{8} - \frac{t}{10}\right)\Big|_0^3$ or 35.825

[+2] 4. The cumulative profit a business has earned is changing at a rate of $r(t)$ dollars per day (where t is the time in days). In the first 30 days, the business earned a cumulative profit of \$1700.

+1 (a) What does $1700 + \int_{30}^{90} r(t)dt$ represent?

$\underbrace{\underset{\text{profit 1st 30 days}}{1700} + \underset{\text{profit days 30 to 90}}{\int_{30}^{90} r(t)dt}}_{\text{cumulative profit for 1st 90 days}}$

+1 (b) What are the units of measure for $1700 + \int_{30}^{90} r(t)dt$?

dollars (NOT dollars per day)

amount not rate

+4

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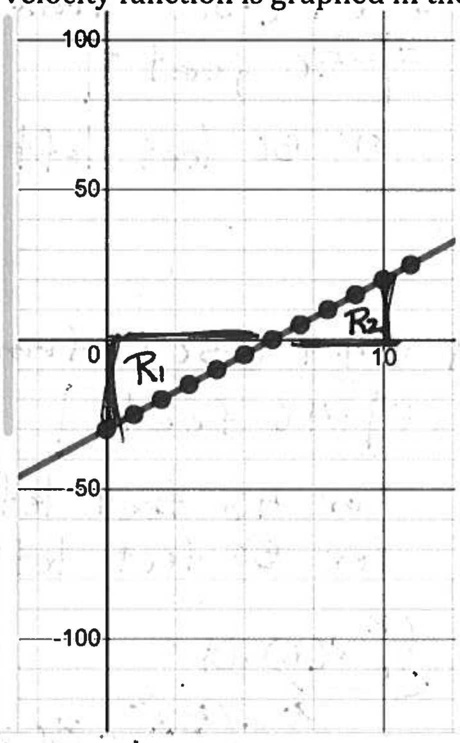
SECTION TWO: A Motion Problem

A particle is moving along the x-axis. Its velocity (in feet per second) is given by $v(t) = 5t - 30$. This velocity function is graphed in the following Desmos worksheet:

$v(t) = 5t - 30$

x_1	$v(x_1)$
0	−30
1	−25
2	−20
3	−15
4	−10
5	−5
6	0
7	5
8	10
9	15
10	20
11	25



[+1]

1. Find the <u>(net) change in position</u> between times $t = 0$ and $t = 10$.

"Net change in position" $= \int_0^{10} (5t-30)\,dt$

$= -(\text{area } R_1) + (\text{area } R_2)$

$= -\left(\tfrac{1}{2}(6)(30)\right) + \left(\tfrac{1}{2}(4)(20)\right)$ *

$= -(90) + 40 = \boxed{-50 \text{ feet}}$

OR $\int_0^{10} (5t-30)\,dt = \left(\frac{5t^2}{2} - 30t\right)\Big|_0^{10} = \frac{5(10)^2}{2} - 30(10)$

$= 250 - 300 = -50$

<u>NOT</u> 130 feet - that is total distance travelled: $\int_0^{10} |5t-30|\,dt$

+1. <u>NOT</u> 5 ft/sec - that is "average acceleration" or "average rate of change of velocity"

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[+1]

2. If we start tracking the particle at time $t = 0$ when it is at the *origin* on the *x-axis,* what is its position at time $t = 10$ *seconds*?

$$0 + \int_0^{10} (5t-30)\,dt = 0 + (-50) \text{ from \#1}$$
$$= \boxed{-50 \text{ feet}}$$

or 50 feet left of origin

[+1]

3. (In a different scenario) if we start tracking the particle at time $t = 0$ when it is at point $(4,0)$ on the *x-axis,* what is its position at time $t = 10$ *seconds*?

$$4 + \int_0^{10} (5t-30)\,dt = 4 + (-50) \text{ from \#1}$$
$$= \boxed{-46 \text{ feet}}$$

or 46 feet left of origin
(or 50 feet left of $(4,0)$)

[+1]

4. Find a formula for the position function $s(t)$ in #3 when $s(0) = 4$.

$$s(x) = \underset{\substack{\uparrow \\ 4}}{s(0)} + \underbrace{\int_0^x v(t)\,dt}_{\left(\frac{5x^2}{2} - 30x\right)}$$

$$= 4 + \left(\frac{5x^2}{2} - 30x\right) \text{ or } \boxed{2.5x^2 - 30x + 4}$$

[+1]

5. Find the average value of the original velocity function on the time interval $[0,10]$.

"average function value"

$$\frac{1}{10-0}\underbrace{\int_0^{10} (5t-30)\,dt}_{\text{from \#1}} = \frac{1}{10}(-50) = \boxed{-5 \text{ ft/sec}}$$

[+1]

6. Is your position function in #4 concave up or concave down?
Justify your answer using calculus concepts.

$\boxed{\text{concave up}}$ "Rate of change of position function is increasing."

because position function is $s(t)$

$s'(t) = v(t)$, velocity function, $= 5t - 30$

$s''(t) = v'(t) = 5$, acceleration function

$s''(t) > 0 \Rightarrow$ so s is concave up

"second derivative of s is positive."

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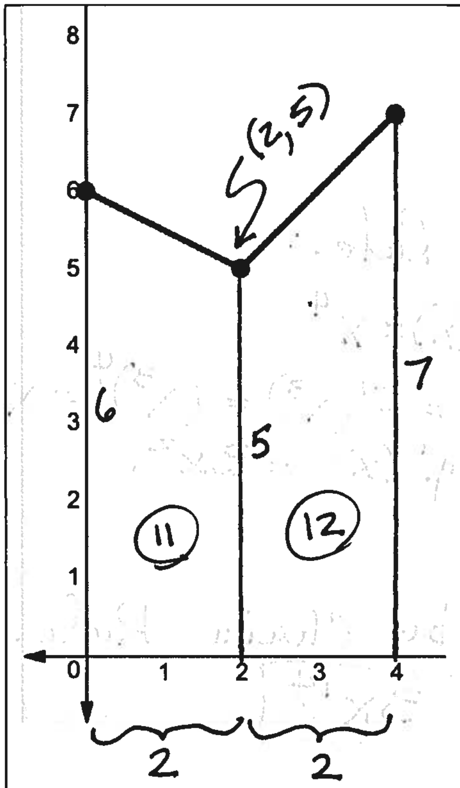
SECTION THREE: Integral Notation & Function Attributes

The portion of the graph of function f on the interval [0, 4] is shown below.
The graph of f contains the points (0,6), (2,5), and (4,7) as shown.



Let $g(x) = \int_0^x f(t)dt$.

[+1] 1. Evaluate: $g(4)$

$\int_0^4 f(t)\,dt$ = area of region

$= 11 + 12 = \boxed{23}$

[+1] 2. Write in simplest form: $\frac{d}{dx}\int_0^x f(t)dt$

$\boxed{f(x)}$ "Second F.T. of C."

$\boxed{+0.9 \text{ for } f(t)}$

[+1] 3. Evaluate: $g'(2)$

$f(2) = \boxed{5}$

[+1] 4. On interval $[0,4]$, where is g decreasing?

g is always increasing

NEVER

[+1] 5. If function h_1 is defined on $[-4,4]$, matches f on $[0,4]$, and is <u>even</u>. Evaluate: $\int_{-4}^{4} h_1(t)dt$.

$23 + 23 = \boxed{46}$

[+1] 6. If function h_2 is defined on $[-4,4]$, matches f on $[0,4]$, and is <u>odd</u>. Evaluate: $\int_{-4}^{4} h_2(t)dt$.

$(-23) + (23) = \boxed{0}$

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SECTION FOUR: Second Fundamental Theorem of Calculus & Chain Rule

[+1] Simplify (without the derivative and integral symbols):

$$\frac{d}{dx}\int_0^{x^3} (t^4)\ dt$$

Using Second FTC and Chain Rule:

Let $F(x) = \int_0^x (t^4)dt$. Then $F'(x) = x^4$

Let $g(x) = x^3$. Then, $F'(g(x)) = F'(x^3) = (x^3)^4 = x^{12}$

and $g'(x) = 3x^2$.

$\int_0^{x^3} (t^4)dt = F(g(x))$

$\frac{d}{dx}\int_0^{x^3} (t^4)dt = F'(g(x))\, g'(x)$ by Chain Rule.

$= (x^{12})(3x^2) = \boxed{3x^{14}}$

Using First FTC (avoids Chain Rule):

$\int_0^{x^3} (t^4)dt = \left(\frac{t^5}{5}\right)\Big|_0^{x^3} = \frac{(x^3)^5}{5} - \frac{0^5}{5} = \frac{x^{15}}{5}$

$\frac{d}{dx}\int_0^{x^3} (t^4)\,dt = \frac{d}{dx}\left(\frac{x^{15}}{5}\right) = \frac{15x^{14}}{5} = \boxed{3x^{14}}$

SCORING:

+1 for $3x^{14}$ with justifying work.

+0.8 for finding $\int_0^{x^3} (t^4)dt = \frac{x^{15}}{5}$

+0.7 for getting close to

+0.5 for $\frac{t^5}{5}$ - good start.

$\boxed{+1}$